Calculation Policy

This policy has been written using the NCETM Calculation Guidance and the DfE Non-Statutory Guidance. This policy should be read in conjunction with the Factual Fluency Progression document.

This policy is to ensure that pupils develop efficient strategies for solving calculations. Efficiency in calculation requires having a variety of mental strategies. Informal methods of recording calculations are an important stage to help children develop fluency with formal methods of recording. Teaching column methods for calculation provides the opportunity to develop both procedural and conceptual fluency. Teachers need to ensure that pupils understand the structure of the mathematics presented in the algorithms, with a particular focus on place value.

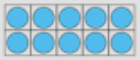
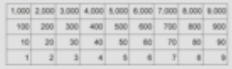
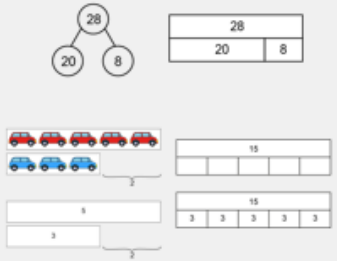
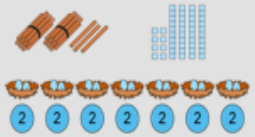
Children's conceptual understanding and fluency is strengthened if they experience concrete, visual, and abstract representations of a concept during a lesson. Moving between the concrete and the abstract helps children to connect abstract symbols with familiar contexts, thus providing the opportunity to make sense of, and develop fluency in the use of, abstract symbols.

Pupils should be able to choose and use efficient calculation methods for addition, subtraction, multiplication and division. They must also have automatic recall of a core set of multiplicative and additive facts to enable them to focus on learning new concepts (see factual fluency progression document).

Non-Statutory Guidance Calculation and Fluency	
Year 1	The main addition and subtraction calculation focus in year 1 is developing fluency in additive facts within 10. Fluency in these facts allows pupils to more easily master addition and subtraction with 2-digit numbers in year 2 and underpins all future additive calculation. Pupils must be fluent in counting in multiples of 2, 5 and 10 by the end of year 1.
Year 2	At first, pupils will use manipulatives, such as tens frames, to understand the strategies for adding and subtracting across 10. However, they should not be using the manipulatives as a tool for finding answers. Pupils should be able to carry out these calculations mentally, using their fluency in complements to 10 and partitioning. Year 2 pupils will need lots of practice to be able to add and subtract across 10 with sufficient fluency to make progress with the year 3 curriculum. They should also continue to practise adding and subtracting within 10. Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract only ones or only tens to/from a two-digit number. Pupils should be able to solve these calculations mentally and be able to demonstrate their reasoning either verbally or with manipulatives or drawings. When adding and subtracting any 2 two-digit numbers, to avoid overload of working memory, pupils should learn how to record the steps using informal written notation or equation sequences. Pupils do not need to learn formal written methods for addition and subtraction in year 2.
Y3, 4, 5 & 6	Representations such as place-value counters and partitioning diagrams, and tens-frames with place-value counters, can be used initially to help pupils understand calculation strategies and make connections between known facts and related calculations. However, pupils should not rely on such representations for calculating. Pupils should be developing fluency in both formal written and mental methods for addition and

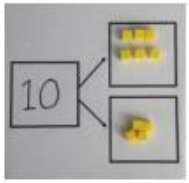
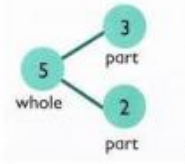
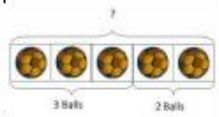
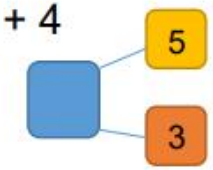
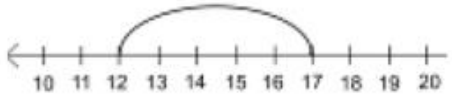
	subtraction and maintain them. Mental methods can include jottings to keep track of calculation, or language structures. Pupils should select the most efficient method to calculate depending on the numbers involved.
Year 3	Mental methods can include jottings to keep track of calculation, or language structures. Pupils should select the most efficient method to calculate depending on the numbers involved. Pupils must be able to add and subtract using columnar addition and subtraction. Pupils should make sensible decisions about how and when to use columnar subtraction.
Year 4	Recall multiplication and division facts up to 12x12 and recognise products in multiplication tables as multiples of the corresponding number. Recall of all multiplication table facts should be the main multiplication calculation focus in year 4. Pupils who leave year 4 fluent in these facts have the best chance of mastering short multiplication in year 5.
Year 5	Pupils who have automatic recall of multiplication table facts and corresponding division facts have the best chance of mastering formal written methods. The facts up to 9 x9 are required for calculation within the 'columns' during application of formal written methods, and all mental multiplicative calculation also depends on these facts. Pupils must be able to multiply using short multiplication and divide a number using a short division.
Year 6	Pupils should make sensible decisions about how and when to use columnar methods. In year 5, pupils learnt to multiply using short multiplication and divide using short division. They should continue to practise this in year 6. Pupils should also be able to represent calculations using the formal written method of long multiplication. For division, pupils should use short or long division as appropriate to calculations. Pupils should learn to check all their calculations with a calculator so that they know how to use one. This will help pupils when they progress to key stage 3.

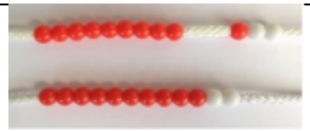
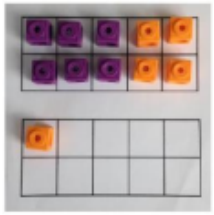
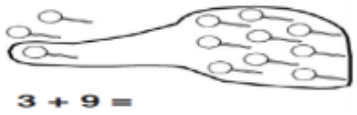
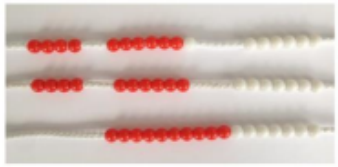
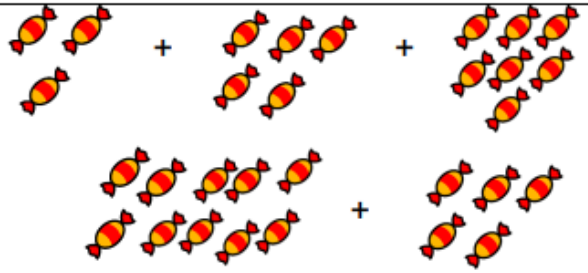
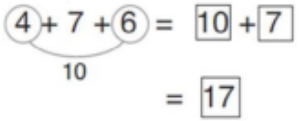
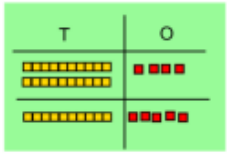
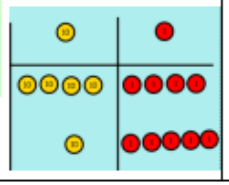
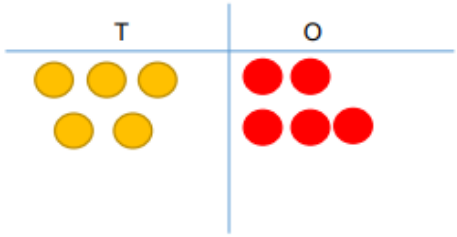
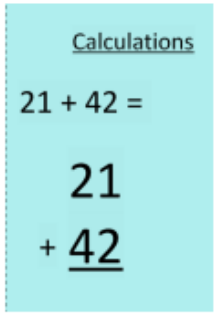
A core set of representations have been selected to expose important mathematical structures and ideas, and make them accessible to pupils. Consistent use of the same representations across year groups help to connect prior learning to new learning.

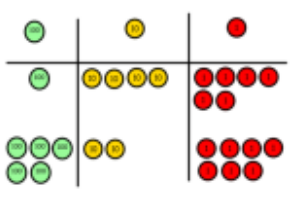
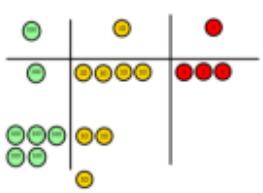
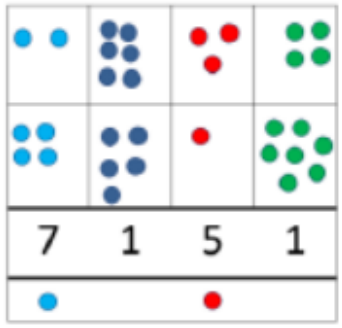
Representation		Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Tens frame	1NPV-1 1AS-1 1 NF-1	2AS-1 2AS-3	3NPV-1 3NF-1 3NF-3	4NPV-1 4NF-3	5NPV-1 5NF-2 5MD-1	6NPV-1
	Number line	1NPV-1 1NPV-2 1NF-2	2NPV-2 2AS-2	3NPV-3 3F-3 3F-4	4NPV-3 4F-1 4F-2 4F-3	5NPV-3 5F-2 5F-3	6NPV-3 6F-1
	Gattegno chart	1NPV-1 1NF-2			4MD-1	5NPV-2 5MD-1	6NPV-1
	Partitioning diagrams including bar models	1AS-1 1AS-2 1NF-1	2NPV-1 2AS-1 2AS-3 2AS-4	3NPV-2 3NPV4 3AS-1 3AS-2 3AS-3 3F-2 3F-4	4NPV-2 4NPV-4 4MD-2 4F-3	5NPV-2 5NPV-4 5F-1	6NPV-4 6AS/MD-4 6F-3
	Groups of units in addition to ones such as Dienes, PV counters		2NPV-1 2AS-3 2AS-4 2MD-1 2MD-2	3AS-2 3MD-1	4MD-2 4F-2	5 NPV-1 5MD-3 5MD-4	6NPV-2

Progression in Calculations

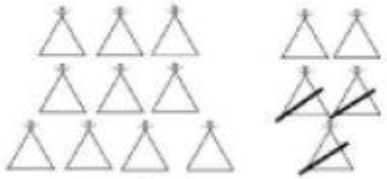
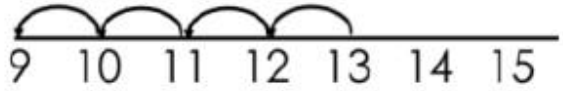
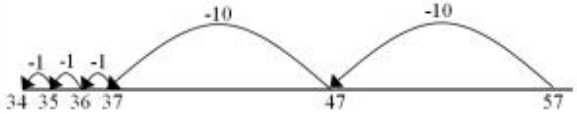
Addition

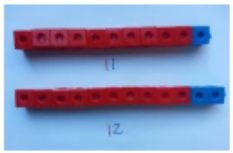
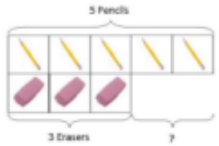
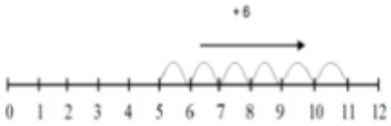
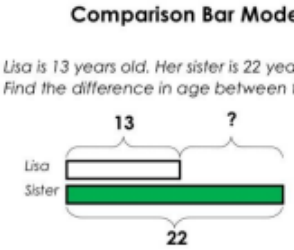
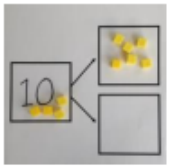
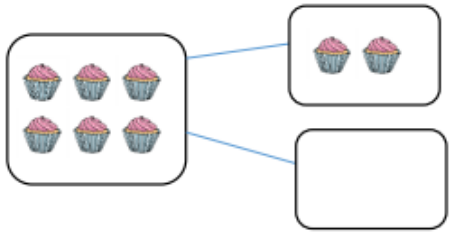
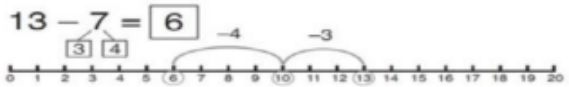
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or in a bar. 	    Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

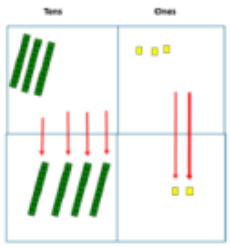
Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10. 9 + 5 = 14 1 4 +1 +4 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	 Calculations 21 + 42 = 21 + 42

Column method- regrouping	Make both numbers on a place value grid.  146 + 527 Add up the units and exchange 10 ones for one 10.  146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  7 1 5 1	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 536 + 85 621 1 1 72.8 + 54.6 127.4 1 1 £23.59 + £7.55 £31.14 23.361 9.080 59.770 + 1.300 93.511 2 1 2

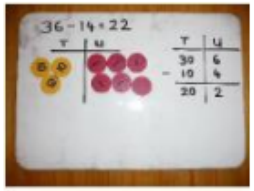
Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -10 -10 -1 -1 -1 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers.  **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 13 ? Lisa Sister 22	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?

<table>
<tr>
<td>Column method without regrouping</td>
<td>


Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.


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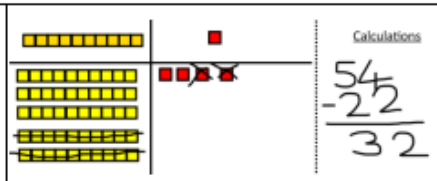
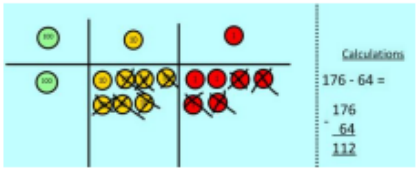


Draw the Base 10 or place value counters alongside the written calculation to help to show working.


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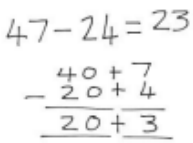
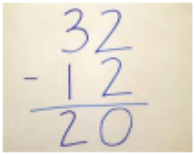


This will lead to a clear written column subtraction.


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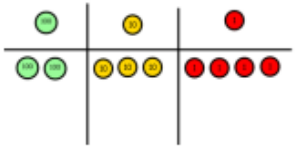
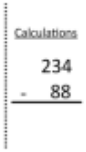
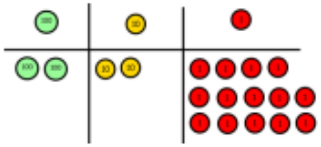
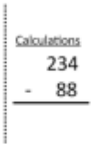
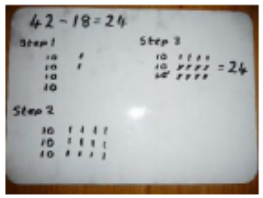
<tr>
<td>Column method with regrouping</td>
<td>
Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters



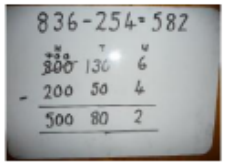
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



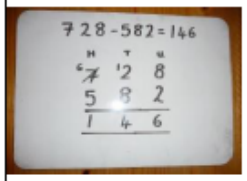
</td>
<td>


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

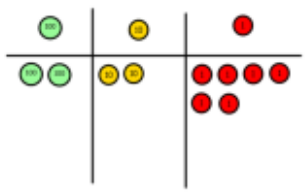
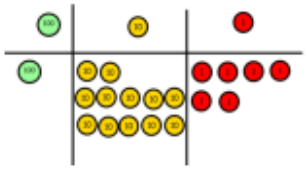


When confident, children can find their own way to record the exchange/regrouping.

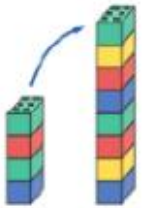
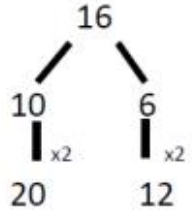
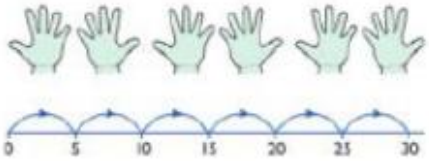
Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.
</td>
<td>


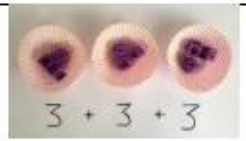
Children can start their formal written method by partitioning the number into clear place value columns.


Moving forward the children use a more compact method.
</td>
</tr>
</table>

	Now I can subtract my ones.  Calculations 234 - 88 Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Calculations 234 - 88 Now I can take away eight tens and complete my subtraction  Calculations 234 - 88 146 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		This will lead to an understanding of subtracting any number including decimals. 5 12 1 2 ~~6~~ ~~3~~ . **0** - 2 6 . 5 2 3 6 . 5

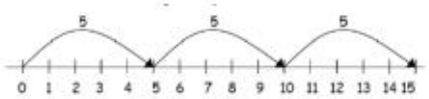
Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number. Double 4 is 8 	 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

<table>
<tr>
<td>Repeated addition</td>
<td> 



Use different objects to add equal groups.</td>
<td>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?

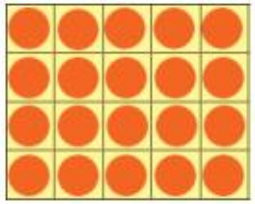
2 add 2 add 2 equals 6


5 + 5 + 5 = 15</td>
<td>Write addition sentences to describe objects and pictures.

2 + 2 + 2 + 2 + 2 = 10</td>
</tr>
<tr>
<td>Arrays- showing commutative multiplication</td>
<td>Create arrays using counters/ cubes to show multiplication sentences.

 </td>
<td>Draw arrays in different rotations to find commutative multiplication sentences.

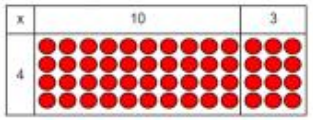
4 × 2 = 8
2 × 4 = 8

2 × 4 = 8
4 × 2 = 8

Link arrays to area of rectangles.</td>
<td>Use an array to write multiplication sentences and reinforce repeated addition.

5 + 5 + 5 = 15
3 + 3 + 3 + 3 + 3 = 15
5 x 3 = 15
3 x 5 = 15</td>
</tr>
</table>

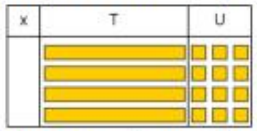
Grid Method

Show the link with arrays to first introduce the grid method.



4 rows of 10
4 rows of 3

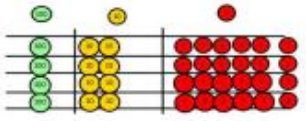
Move on to using Base 10 to move towards a more compact method.



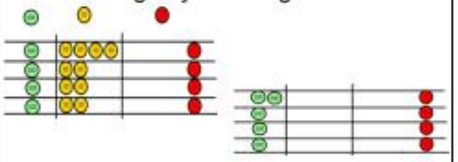
4 rows of 13

Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.

Fill each row with 126.



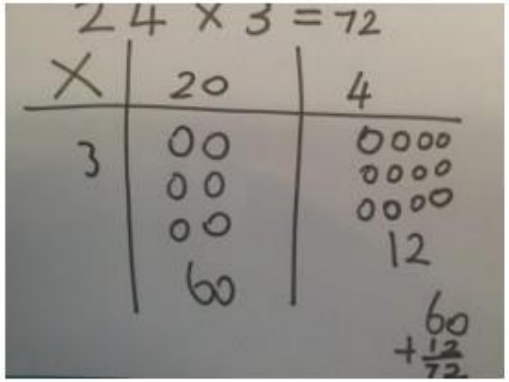
Add up each column, starting with the ones making any exchanges needed.



Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

×	30	5
7	210	35

210 + 35 = 245

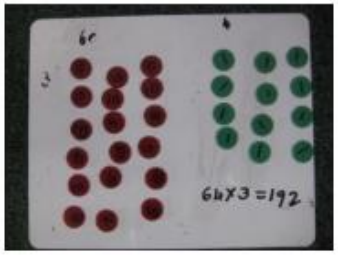
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

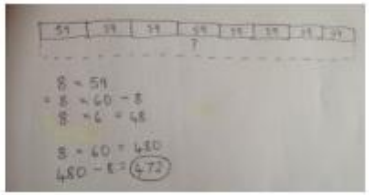
Column multiplication

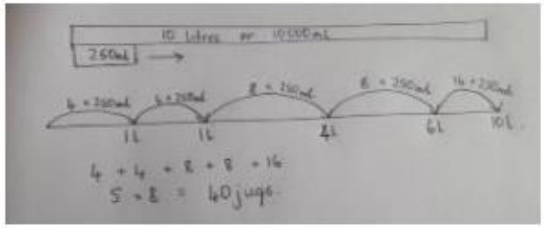
Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.





Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

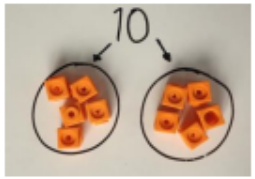
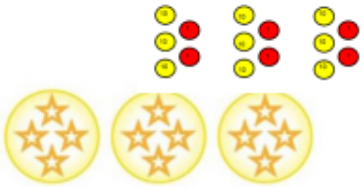
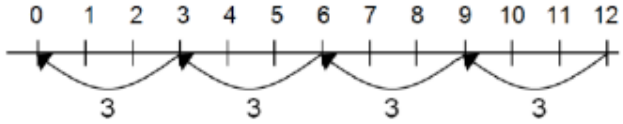
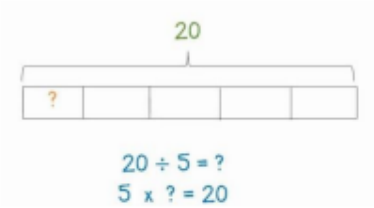
```
  32
x 24
   8   (4 x 2)
 120   (4 x 30)
  40   (20 x 2)
 600   (20 x 30)
 768
```

```
    74
  x 63
    12
   210
   240
+ 420
  4662
```

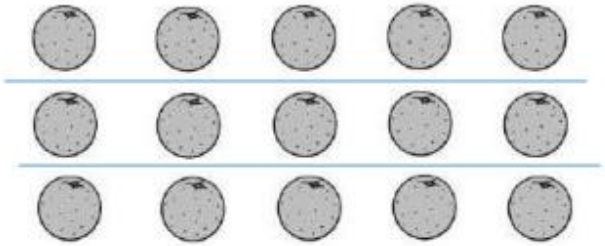
This moves to the more compact method.

```
  2 3 1
  1342
x   18
13420
10736
24156
  1
```

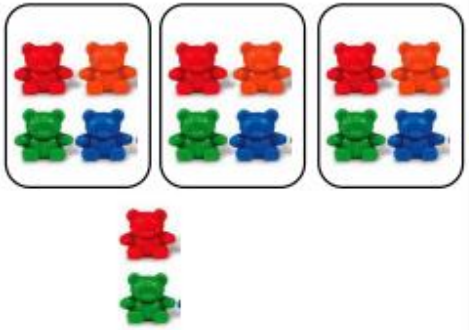
Division

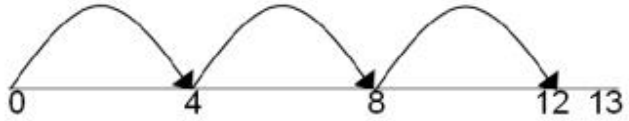
Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

<table>
<tr>
<td>Division within arrays</td>
<td> 
Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg 15 ÷ 3 = 5 5 x 3 = 15
15 ÷ 5 = 3 3 x 5 = 15</td>
<td> 
Draw an array and use lines to split the array into groups to make multiplication and division sentences.</td>
<td>Find the inverse of multiplication and division sentences by creating four linking number sentences.

7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7</td>
</tr>
<tr>
<td>Division with a remainder</td>
<td>14 ÷ 3 =
Divide objects between groups and see how much is left over

  </td>
<td>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.

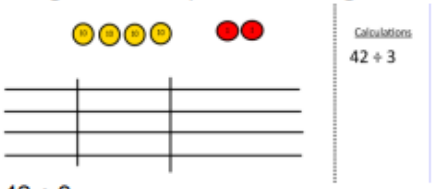
Draw dots and group them to divide an amount and clearly show a remainder.

 
</td>
<td>Complete written divisions and show the remainder using r.

 
</td>
</tr>
</table>

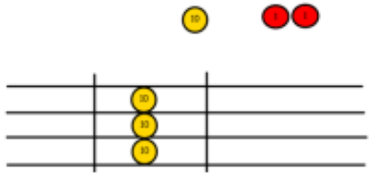
Short division

Use place value counters to divide using the bus stop method alongside

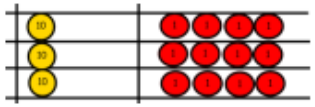


42 ÷ 3=

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

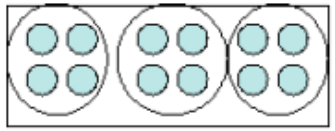


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 4\overline{)872} \\ \underline{8} \\ 07 \\ \underline{4} \\ 32 \\ \underline{32} \\ 0 \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86\ r\ 2 \\ 5\overline{)432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35\overline{)511.0} \\ \underline{35} \\ 161 \\ \underline{140} \\ 210 \\ \underline{210} \\ 0 \end{array}$$

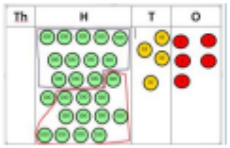
Long division

2544 ÷ 12
How many groups of 12 thousands do we have?
None

Exchange 2 thousand for 20 hundreds.

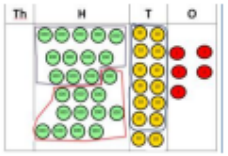
$$\begin{array}{r} 0 \\ 12\overline{)2544} \end{array}$$

How many groups of 12 are in 25 hundreds? 2 groups. Circle them.
We have grouped 24 hundreds so can take them off and we are left with one.



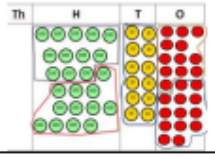
$$\begin{array}{r} 0\,2 \\ 12\overline{)2544} \\ \underline{24} \\ 1 \end{array}$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2



$$\begin{array}{r} 0\,2\,1 \\ 12\overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2



$$\begin{array}{r} 0\,2\,1\,2 \\ 12\overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books.

Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.

$$\begin{array}{r} 0\,3\,1\,8\ r\,5 \\ 20\overline{)6\,3\,6\,5} \\ \underline{-6\,0} \\ 3\,6 \\ \underline{-2\,0} \\ 1\,6\,5 \\ \underline{-1\,6\,0} \\ 5 \end{array}$$